FUZZY REJECTIVE CORE OF SATIATED ECONOMIES WITH UNBOUNDED CONSUMPTION SETS

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Abstract. For an exchange economy, under assumptions which did not bring about the existence of quasiequilibrium with dividends as yet, we prove the nonemptiness of the fuzzy rejective core. Then, via Konovalov (1998, 2005)'s equivalence result, we solve the equilibrium existence problem. Under the same assumptions, we show in a last section the existence of a Walrasian quasiequilibrium under a weak non-satiation condition which differs from the weak non-satiation assumption introduced by Allouch–Le Van (2009). This result, which fits with exchange economies whose consumers' utility functions are not assumed to be upper semicontinuous, complements the one obtained by Martins-da-Rocha and Monteiro (2009).

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1. Introduction

In this note, we consider an exchange economy $\mathcal{E} = \big((X_i, u_i, e_i)_{i\in I}\big)$ defined on the commodity space $\mathbb{R}^\ell$. Each of a finite set I of consumers has a consumption set $X_i \subset \mathbb{R}^\ell$, an initial endowment $e_i \in \mathbb{R}^\ell$ and transitive and complete preferences on X_i represented by a utility function $u_i\colon X_i \to \mathbb{R}$. We normalize utility functions by requiring for each i, $u_i(e_i) = 0$. As usual,

$$\mathcal{A}(\mathcal{E}) := \left\{ x = (x_i)_{i\in I} \in \prod_{i\in I} X_i \colon \sum_{i\in I} x_i = \sum_{i\in I} e_i \right\}$$

denotes the set of all attainable allocations of the economy $\mathcal{E}$. We let

$$\mathcal{A}_{IR}(\mathcal{E}) = \{x \in \mathcal{A}(\mathcal{E}),\ \text{s.t.}\ \ 0 \le u_i(x_i)\ \forall i \in I\}$$

$$\mathcal{U} = \big\{v = (v_i)_{i\in I} \in \mathbb{R}^I \colon \exists x \in \mathcal{A}(\mathcal{E}),\ \text{s.t.}\ \ 0 \le v_i \le u_i(x_i)\ \forall i \in I\big\}$$

be respectively the set of all individually rational attainable allocations and the *individually rational utility set*, sometimes called *utility set*. We will occasionally denote by $\mathcal{A}_i$ the projection on X_i of $\mathcal{A}_{IR}(\mathcal{E})$.

Definition 1.1. *A couple (p, x) where $0 \ne p \in \mathbb{R}^\ell$ and $x = (x_i)_{i\in I} \in \mathcal{A}(\mathcal{E})$ is a (**Walrasian**) **quasiequilibrium** of $\mathcal{E}$ if for each $i \in I$,*

$$p \cdot x_i = p \cdot e_i \ \text{and}\ \ u_i(x'_i) > u_i(x_i) \implies p \cdot x'_i \ge p \cdot e_i.$$

It is an ***equilibrium*** *if* $u_i(x'_i) > u_i(x_i)$ *actually implies* $p \cdot x'_i > p \cdot e_i$

The possibility that the current assumptions do not imply local no-satiation of preferences at each consumption component of equilibrium has motivated the following definition, going back to different authors in different contexts, formalized with a different name by Mas-Colell (1992).

Definition 1.2. *A couple* (p, x) *of a price* $p \in \mathbb{R}^\ell$ *and of an attainable allocation* $x = (x_i)_{i \in I} \in \mathcal{A}(\mathcal{E})$ *is a* ***quasiequilibrium with dividends (slacks)*** *if for each* $i \in I$*, there exists* $m_i \in \mathbb{R}_+$ *such that*

$$p \cdot x_i \leq p \cdot e_i + m_i \text{ and } u_i(x'_i) > u_i(x_i) \implies p \cdot x'_i \geq p \cdot e_i + m_i.$$

It is an ***equilibrium*** *(with dividends* $(m_i)_{i \in I}$ *) if* $u_i(x'_i) > u_i(x_i)$ *actually implies* $p \cdot x'_i > p \cdot e_i + m_i$*.*

It is worth noticing that, in the previous definition, consumers are not anymore required to bind their budget constraint at quasiequilibrium. If all dividends are not equal to 0, the fact that x is an attainable allocation does not imply such an equality between quasiequilibrium expenditure and revenue that only local non-satiation at quasiequilibrium could explain. Moreover, a null equilibrium price is compatible with the previous definition. It simply implies that all consumers are satiated at equilibrium.

On the other hand, let us now introduce the core notions which will allow to deduce the existence of equilibria with dividends from the core equivalence theorem proved by Konovalov (2005).

Definition 1.3. *A coalition* S ***rejects*** *an attainable allocation* $x \in \mathcal{A}(\mathcal{E})$ *if there exists a partition* $S = S^1 \cup S^2$ *and consumption bundles* $y_i \in X_i$*,* $i \in S$*, such that*

(a) $\sum_{i \in S} y_i = \sum_{i \in S^1} e_i + \sum_{i \in S^2} x_i$*,*
(b) $u_i(y_i) > u_i(x_i)$ $\forall i \in S$*.*

Following Aubin (1979), a non-null element $t = (t_i)_{i \in I}$ of the set $[0, 1]^I$ is called a **fuzzy coalition**. Its ith component is interpreted as the rate of participation of agent i in the fuzzy coalition. Obviously, a coalition $S \subset I$ can be seen as a fuzzy coalition with rates of participation equal to 0 or 1.

Definition 1.4. *A fuzzy coalition* $t = (t_i)_{i \in I}$ ***rejects*** *an attainable allocation* $x \in \mathcal{A}(\mathcal{E})$ *if there exists* t^1, t^2 *in* $[0, 1]^I$ *and* $y \in \prod_{i \in I} X_i$ *such that* $t = t^1 + t^2$ *and*

(a) $\sum_{i \in I} t_i y_i = \sum_{i \in I} t^1_i e_i + \sum_{i \in I} t^2_i x_i$*,*
(b) $u_i(y_i) > u_i(x_i)$ $\forall i \colon t_i > 0$*.*

Definition 1.5. *An attainable allocation* $x \in \mathcal{A}(\mathcal{E})$ *is an element of the* ***rejective core*** $\mathcal{C}_r(\mathcal{E})$ *(resp. is an element of the* ***fuzzy rejective core*** $\mathcal{C}^f_r(\mathcal{E})$*) if it cannot be rejected by a coalition (resp. a fuzzy coalition). It is an element of the* ***Edgeworth rejective core*** $\mathcal{C}^e_r(\mathcal{E})$ *if it cannot be rejected by a fuzzy coalition with rational rates* t_i*,* t^1_i*,* t^2_i *of participation.*

If, in definitions 1.3 and 1.4, $S^2 = \emptyset$ or if $t^2 = 0$, the previous definitions coincide with the standard definitions of blocking, so that $\mathcal{C}_r(\mathcal{E}) \subset \mathcal{C}(\mathcal{E})$, $\mathcal{C}^f_r(\mathcal{E}) \subset \mathcal{C}^f(\mathcal{E})$, $\mathcal{C}^e_r(\mathcal{E}) \subset \mathcal{C}^e(\mathcal{E})$. It easily follows from the definitions that an attainable allocation of the rejective core (resp. fuzzy rejective core, Edgeworth rejective core) is individually rational, that is belongs to $\mathcal{A}_{IR}(\mathcal{E})$. Moreover, if satiation at all components of an attainable allocation is not ruled out, such an attainable allocation is a trivial element of $\mathcal{C}_r(\mathcal{E})$ (resp. $\mathcal{C}^e_r(\mathcal{E})$, $\mathcal{C}^f_r(\mathcal{E})$). it is also a trivial equilibrium with dividends allocation with any null or non-null equilibrium price.

The main purpose of this note is to give sufficient assumptions on the economy $\mathcal{E}$ to guarantee the non-emptiness of its fuzzy rejective core. This will be done in Section 2. In Section 3, we will deduce from this result new conditions for the existence of quasiequiliibrium with dividends in $\mathcal{E}$. Under the same assumptions, we will show in a last section the existence of a Walrasian quasiequilibrium under a weak non-satiation condition which differs from the weak non-satiation assumption introduced by Allouch–Le Van (2009).

2. The non-emptiness of the fuzzy rejective core

We will maintain on $\mathcal{E}$ the following assumptions.

- **A.1:** For each i, X_i is convex, containing e_i;
- **A.2:** For each i, u_i is strictly quasi-concave;
- **A.3:** For each i, the set $P_i(x_i) := \{y_i \in X_i : u_i(y_i) > u_i(x_i)\}$ is open in X_i at every attainable and individually rational consumption vector;[1]
- **A.4:** For each i, $e_i \in \operatorname{int} X_i$.

In addition, when consumption sets are not bounded below, a compactness assumption should limit the arbitrage possibilities for consumers. We borrow this assumption from Martins-da-Rocha and Monteiro (2009). Let us denote for each $i \in I$, $S_i = \operatorname{argmax}\{u_i(x_i) : x_i \in X_i\}$ the set of satiation points of u_i on X_i, and for every attainable allocation $x \in \mathcal{A}(\mathcal{E})$, $S(x) = \{i \in I : x_i \in S_i\}$, $N(x) = I \setminus S(x)$.

Definition 2.1. *The individually rational utility set $\mathcal{U}$ is* ***strongly compact*** *if for every sequence x^ν in $\mathcal{A}_{IR}(\mathcal{E})$ there exists an attainable allocation $y \in \mathcal{A}(\mathcal{E})$ and a subsequence x^{ν_k} satisfying*

(a) $\forall i \in I$, $u_i(y_i) \geq \lim_{k\to\infty} u_i(x_i^{\nu_k})$,

(b) $\forall i \in I$, $\lim_{k\to\infty} \frac{\mathbf{1}_{S_i}(x_i^{\nu_k})}{1+\|x_i^{\nu_k}\|^2}(y_i - x_i^{\nu_k}) = 0$.[2]

As explained by Martins-da-Rocha and Monteiro, condition *b.* means that for the sequence $x_i^{\nu_k}$, only three cases can occur:

(1) the subsequence $x_i^{\nu_k}$ is unbounded,
(2) the subsequence $x_i^{\nu_k}$ converges to y_i,
(3) for k large enough, $x_i^{\nu_k}$ is not a satiation point.

The two following implications are proved in Martins-da-Rocha–Monteiro (2009) where are also provided examples showing that the reverse implications are not true.

- If the utility functions u_i are upper semicontinuous,[3] compactness of $\mathcal{A}_{IR}(\mathcal{E})$ implies the strong compactness of $\mathcal{U}$,
- strong compactness of $\mathcal{U}$ implies its compactness.

It is well known that if $\mathcal{U}$ is compact, the economy $\mathcal{E}$ has a (strong) Pareto optimum[4] which is also individually rational. Strong compactness of $\mathcal{U}$ allows to prove the following sharper result to be used later (see the discussion between Claim 2 and Claim 3 in the proof of Proposition 2.2). For

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[1] As noticed several times in the literature, this assumption is not necessary if in **A.2**, the functions u_i are assumed to be concave.

[2] If A is a subset of $\mathbb{R}^\ell$, for $z \in \mathbb{R}^\ell$, $\mathbf{1}_A(z)$ is defined by $\mathbf{1}_A(z) = z$ if $z \in A$, $\mathbf{1}_A(z) = 0$ otherwise.

[3] It is obvious that compactness properties of $\mathcal{U}$ subsume some upper semicontinuity properties of utility function. When the compactness or the strong compactness of $\mathcal{U}$ is directly assumed, according to the results to be got, it may be not necessary to assume in addition that the utility functions are upper semicontinuous.

[4] That is, $x^* \in \mathcal{A}(\mathcal{E})$ such that there is no $y \in \mathcal{A}(\mathcal{E})$ with $u_i(y_i) \geq u_i(x_i^*)$ for each $i \in I$ and $u_{i_0}(y_{i_0}) > u_{i_0}(x_{i_0}^*)$ for some $i_0 \in I$.

$\overline{x} \in \mathcal{A}_{IR}(\mathcal{E})$, let us define the following subset of $\mathcal{U}$

$$\mathcal{U}(\overline{x}) = \left\{ v = (v_i)_{i\in I} \in \mathbb{R}^I : \exists x \in \mathcal{A}(\mathcal{E}), \text{ s.t. } \begin{array}{c} 0 \leq v_i \leq u_i(x_i)\ \forall i \in I, x_i = \overline{x}_i\ \forall i \in S(\overline{x}), \\ \text{and } u_i(x_i) \geq u_i(\overline{x}_i)\ \ \forall i \in N(\overline{x}) \end{array} \right\}.$$

Proposition 2.1. *Assume that $\mathcal{U}$ is nonempty and strongly compact. Then for $\overline{x} \in \mathcal{A}_{IR}(\mathcal{E})$, the set $\mathcal{U}(\overline{x})$ has a maximal element.*

Proof. it suffices to verify that $\mathcal{U}(\overline{x})$ is closed. Let v^ν a sequence of elements of $\mathcal{U}(\overline{x})$ converging to v. There exists a sequence x^ν of elements of $\mathcal{A}(\mathcal{E})$ such that $0 \leq v_i^\nu \leq u_i(x_i^\nu)$ $\forall i \in I$ and $x_i^\nu = \overline{x}_i$ $\forall i : \overline{x}_i \in S_i, u_i(x_i^\nu) \geq u_i(\overline{x}_i)$ $\ \forall i : \overline{x}_i \notin S_i$. By strong compactness of $\mathcal{U}$, the exists a subsequence $x_I^{\nu_k}$ and $y \in \mathcal{A}(\mathcal{E})$ satisfying for all $i \in I$, $u_i(y_i) \geq \lim_{k\to\infty} ui(x_i^{\nu_k})$ and one of the three properties above. It then follows that $y_i = \overline{x}_i$ if $\overline{x}_i \in S_i$ and $u_i(y_i) \geq u_i(\overline{x}_i)$ if $\overline{x}_i \notin S_i$, thus that $v \in \mathcal{U}(\overline{x})$. ■

We will keep in mind that if $v^* = \big(u_i(x_i^*)\big)_{i\in I}$ is a maximal element of $\mathcal{U}(\overline{x})$ then $S(\overline{x}) \subset S(x^*)$.

From now on, we add to the assumptions **A.1** – **A.4** on the economy $\mathcal{E}$ the following compactness assumption.

A.5: $\mathcal{U}$ is strongly compact.

The next proposition is the main result of the note.

Proposition 2.2. *Under Assumptions **A.1** – **A.5**, the economy $\mathcal{E}$ has a non-empty fuzzy rejective core.*

Proof. As Le Van–Minh (2007), given some $\mu > 0$ and $\delta = (\delta_i)_{i\in I} \in \mathbb{R}^I_{++}$, we associate to the economy $\mathcal{E}$, the economy

$$\widehat{\mathcal{E}} = \big((\widehat{X}_i, \widehat{u}_i, \widehat{e}_i)_{i\in I}\big)$$

defined as follows. For each $i \in I$, $\widehat{X}_i = X_i \times \mathbb{R}_+$, $\widehat{e}_i = (e_i, \delta_i)$, and, for any $(x_i, d_i) \in \widehat{X}_i$,

$$\widehat{u}_i(x_i, d_i) = \begin{cases} u_i(x_i) & \text{if, in } \mathcal{E},\ x_i \notin S_i \\ u_i(x_i) + \mu d_i & \text{if, in } \mathcal{E},\ x_i \in S_i \end{cases}$$

Without loss of generality, we can assume that no e_i is satiation point of u_i on X_i, so that $\widehat{u}_i(e_i, \delta_i) = 0$.[5] For the economy $\widehat{\mathcal{E}}$, the sets $\mathcal{A}(\widehat{\mathcal{E}})$, $\mathcal{A}_{IR}(\widehat{\mathcal{E}})$, $\mathcal{U}$ are defined as in $\mathcal{E}$. In particular,

$$\widehat{\mathcal{U}} = \{v = (v_i)_{i\in I} : \exists (x, d) = (x_i, d_i)_{i\in I} \in \mathcal{A}(\widehat{\mathcal{E}}), \text{ s.t. } \ 0 \leq v_i \leq \widehat{u}_i(x_i, d_i)\ \forall i \in I\}.$$

The proposition will be proved through a sequence of claims. Two first claims will verify the conditions, stated in Proposition 4.1 of Allouch–Florenzano (2004), for existence of Edgeworth equilibria in $\widehat{\mathcal{E}}$. Then, the following claims will show first that to an Edgeworth equilibrium of $\widehat{\mathcal{E}}$ can be associated an element of the Edgeworth rejective core of $\mathcal{E}$, which is, in view of Assumption **A.3**, an element of the fuzzy rejective core of $\mathcal{E}$.

Claim 1. *For each i, $\widehat{X}_i$ is convex, containing $\widehat{e}_i$. Moreover, the function $\widehat{u}_i$ is strictly quasiconcave.*

The claim is proved in Le Van–Minh (2007).

Claim 2. *The set $\widehat{U}$ is compact.*

[5] A consumer who is satiated at e_i does not play any role in the rejection process.

Let $(v^\nu) = \big((v_i^\nu)_{i\in I}\big)$ be a sequence of elements of $\widehat{U}$. By definition, there exists a sequence $\big((x_i^\nu, d_i^\nu)_{i\in I}\big)$ of elements of $\mathcal{A}_{IR}(\widehat{E})$ such that

$$0 \leq v_i^\nu \leq \begin{cases} u_i(x_i^\nu) + \mu d_i^\nu & \text{if } x_i^\nu \text{ is a satiation point} \\ u_i(x_i^\nu) & \text{if } x_i^\nu \text{ is not a satiation point} \end{cases}$$

Using the strong compactness of $\mathcal{U}$, there exists $y = (y_i)_{i\in I} \in \mathcal{A}(\mathcal{E})$and, for each $i \in I$, subsequences $(x_i^{\nu_k})$ satisfying the conditions of Definition 2.1, together with subsequences $(v_i^{\nu_k})$ converging to v_i, $(d_i^{\nu_k})$ converging to $d_i \in \mathbb{R}_+$ such that $(y_i, d_i)_{i\in I} \in \mathcal{A}(\widehat{E})$.
Passing to limit in the above relation, distinguishing whether y_i is or not a satiation point, and observing that if y_i is not a satiation point then for k large enough, $x_i^{\nu_k}$ is not a satiation point, we get for each $i \in I$

$$0 \leq v_i \leq \begin{cases} u_i(y_i) + \mu d_i & \text{if } y_i \text{ is a satiation point} \\ u_i(y_i) & \text{if } y_i \text{ is not a satiation point} \end{cases}$$

so that $0 \leq v_i \leq \widehat{u}_i(y_i, d_i)$, which proves that $v = (v_i)_{i\in I} \in \widehat{U}$.

In view of the previous claims, the economy $\widehat{\mathcal{E}}$ satisfies Assumptions **A.1**–**A.3** of Theorem 3.1 in Allouch–Florenzano (2004), thus has an Edgeworth equilibrium $(\overline{x}, \overline{d})$.

At this stage, it is worthwhile noticing that in $\widehat{\mathcal{E}}$, the set

$$\widehat{P}_i(\overline{x}_i, \overline{d}_i) = \begin{cases} P_i(\overline{x}_i) \times \mathbb{R}_+ & \text{if } \overline{x}_i \notin S_i \\ S_i \times \{d_i > \overline{d}_i\} & \text{otherwise} \end{cases}$$

is not necessarily open in $\widehat{X}_i$. We thus cannot infer from the previous conclusion that $(\overline{x}, \overline{d})$ belongs to the fuzzy core of $\widehat{\mathcal{E}}$. We now indicate how to circumvent this difficulty.

If, in view of Proposition 2.1, $v^* = \big(u_i(x_i^*)\big)_{i\in I}$ is a maximal element of $\mathcal{U}(\overline{x})$, the reader will verify that, in the economy $\widehat{\mathcal{E}}$, $\widehat{u}_i(x_i^*, \overline{d}_i) \geq \widehat{u}_i(\overline{x}_i, \overline{d}_i)$ $\forall i \in I$, and thus that $(x^*, \overline{d})$ is likewise an element of $\mathcal{C}^e(\widehat{\mathcal{E}})$. In the following claim, we intend to show that $x^* = (x_i^*)_{i\in I} \in \mathcal{C}_r^e(\mathcal{E})$, and assume from now on, by way of contradiction, that there exists t^1, t^2 in $([0,1] \cap \mathbb{Q})^I)$ and $y \in \prod_{i\in I} X_i$ such that if $t = t^1 + t^2 \neq 0$ then

(a) $\sum_{i\in I} t_i y_i = \sum_{i\in I} t_i^1 e_i + \sum_{i\in I} t_i^2 x_i^*$,
(b) $u_i(y_i) > u_i(x_i^*)$ $\forall i\colon t_i > 0$.

Taking in account Proposition 1 in the appendix, there is no loss of generality to assume that $t_i = 0$ if and only if $i \in S(x^*)$.

Claim 3. $x^* \in \mathcal{C}_r^e(\mathcal{E})$.

Setting, as Konovalov (2005), $t^2_{i\max} = \max_i t_i^2$, we will distinguish two cases.

Case1: $t^2_{i\max} = 0$. In this case, $\sum_{i\in I} t_i y_i = \sum_{i\in I} t_i e_i$ and $\sum_{i\in I} t_i(y_i, \delta_i) = \sum_{i\in I} t_i(e_i, \delta_i)$. As $\widehat{u}_i(y_i, \delta_i) > \widehat{u}_i(x_i^*, \overline{d}_i)$ $\forall i\colon t_i > 0$, the coalition t blocks the allocation $(x^*, \overline{d})$, which contradicts the fact that $(x^*, \overline{d})$ is an Edgeworth equilibrium of $\widehat{\mathcal{E}}$.

Case 2: $t^2_{i\max} > 0$. We first remark that, in this case, there exists i_0 such that $t^1_{i_0} > 0$. Indeed, if not, from $\sum_{i\in I} t_i^2 y_i = \sum_{i\in I} t_i^2 x_i^*$ with $u_i(y_i) > u_i(x_i^*)$ $\forall i\colon t_i^2 > 0$, we easily deduce: $\sum_{i\in I} t^2_{i\max}\big(\frac{t_i^2}{t^2_{i\max}} y_i + \frac{t^2_{i\max} - t_i^2}{t^2_{i\max}} x_i^*\big) = \sum_{i\in I} t^2_{i\max} e_i$ with if $z_i = \frac{t_i^2}{t^2_{i\max}} y_i + \frac{t^2_{i\max} - t_i^2}{t^2_{i\max}} x_i^*$, $u_i(z_i) > u_i(x_i^*)$

if $t_i^2 > 0$, $z_i = x_i^*$ if $t_i^2 = 0$. As $t_{i\,\max}^2 > 0 \implies z = (z_i)_{i\in I} \in \mathcal{A}(\mathcal{E})$, recalling that $S(\overline{x}) \subset S(x^*)$, it follows that $\big(u_i(z_i)\big)_{i\in I} \in \mathcal{U}(\overline{x})$, which contradicts the definition of x^*.

In view of this remark, using

$$\sum_{i\in I} t_i y_i = \sum_{i\in I} t_i^1 e_i + \sum_{i\in I} t_i^2 x_i^* \tag{2.1}$$

and

$$0 < \sum_{i\in I} t_i^1 \delta_i + \sum_{i\in I} t_i^2 \overline{d}_i \tag{2.2}$$

we can choose for $i \in S(x^*)$ $d_i > \overline{d}_i$ and rational τ_i^2 such that

$$\sum_{i\in S(x^*)} \tau_i^2 (d_i - \overline{d}_i) = \sum_{i\in I} t_i^1 \delta_i + \sum_{i\in I} t_i^2 \overline{d}_i \tag{2.3}$$

with $0 < \tau_i^2 < t_{i\,\max}^2$.

Let in $\widehat{\mathcal{E}}$, the allocation $z = (z_i)_{i\in I}$ with $z_i = \begin{cases} (y_i, 0) & \text{if } i \notin S(x^*) \\ (x_i^*, d_i) & \text{if } i \in S(x^*) \end{cases}$.
Setting $\tau_i^2 = 0$ for $i \in S(x^*)$, $\tau_i^1 = t_i^1$ and $\tau_i^2 = t_i^2$ for $i \notin S(x^*)$, it is easily verified that

$$\sum_{i\in I} (\tau_i^1 + \tau_i^2) z_i = \sum_{i\in I} t_i^1 (e_i, \delta_i) + \sum_{i\in I} \tau_i^2 (x_i^*, \overline{d}_i) \tag{2.4}$$

and that the fuzzy coalition $\tau = \tau^1 + \tau^2$ rejects with z the allocation $(x^*, \overline{d})$. Notice, in addition, that $\tau_i > 0$ $\forall i \in I$.

From (2.4), using the attainability of the allocation $\big(x_i^*, \overline{d}_i\big)_{i\in I}$, we successively deduce:

$$\sum_{i\in I} t_i^1 z_i + \sum_{i\in I} \tau_i^2 z_i + \sum_{i\in I} (t_{i\,\max}^2 - \tau_i^2)(x_i^*, \overline{d}_i) = \sum_{i\in I} t_i^1 (e_i, \delta_i) + \sum_{i\in I} t_{i\,\max}^2 (x_i^*, \overline{d}_i) \tag{2.5}$$

$$\sum_{i\in I} t_i^1 (y_i, 0) + \sum_{i\in I} t_{i\,\max}^2 \Big(\frac{\tau_i^2}{t_{i\,\max}^2} z_i + (1 - \frac{\tau_i^2}{t_{i\,\max}^2})(x_i^*, \overline{d}_i)\Big) = \sum_{i\in I} (t_i^1 + t_{i\,\max}^2)(e_i, \delta_i) \tag{2.6}$$

Set

$$z_i' = \frac{\tau_i^2}{t_{i\,\max}^2} z_i + (1 - \frac{\tau_i^2}{t_{i\,\max}^2})(x_i^*, \overline{d}_i),$$

$$z_i'' = \frac{t_i^1}{t_i^1 + t_{i\,\max}^2} (y_i, 0) + \frac{t_{i\,\max}^2}{t_i^1 + t_{i\,\max}^2} z_i'.$$

For every $i \in I$, let us first assume that $\tau_i^2 > 0$. Then, either $i \in S(x^*)$, $z_i = (x_i^*, d_i)$, and $\widehat{u}_i(z_i') > \widehat{u}_i(x_i^*, \overline{d}_i)$ follows from the strict monotonicity of $\widehat{u}_i$ relative to its second argument when $i \in S(x^*)$ and its strict quasi-concavity; or $i \notin S(x^*)$, $z_i = (y_i, 0)$, $\widehat{u}_i(z_i) > \widehat{u}_i(x_i^*, \overline{d}_i)$ and $\widehat{u}_i(z_i') > \widehat{u}_i(x_i^*, \overline{d}_i)$ by strict quasi-concavity of $\widehat{u}_i$. In both cases, whether $t_i^1 > 0$ or not, $\widehat{u}_i(z_i'') > u_i(x_i^*, \overline{d}_i)$.

Let us now assume that $\tau_i^2 = 0$. Then, $i \notin S(x^*)$, $\tau_i^1 = t_i^1 > 0$, $z_i' = (x_i^*, \overline{d}_i)$, $\widehat{u}_i(y_i, 0) > \widehat{u}_i(x_i^*, \overline{d}_i)$, and $\widehat{u}_i(z_i'') > u_i(x_i^*, \overline{d}_i)$ follows from the strict quasi-concavity of $\widehat{u}_i$.

Summarizing,

$$\sum_{i\in I}\frac{t_i^1+t_{i\,\max}^2}{\sum_{j\in J}(t_j^1+t_{i\,\max}^2)}z_i''=\sum_{i\in I}\frac{t_i^1+t_{i\,\max}^2}{\sum_{j\in J}(t_j^1+t_{i\,\max}^2)}(e_i,\delta_i)$$

with $\widehat{u}_i(z_i'') > \widehat{u}_i(x_i^*, \overline{d}_i)$ $\forall i \in I$, which contradicts $(x^*, \overline{d}) \in \mathcal{C}^e(\widehat{\mathcal{E}})$. The proof of Claim 3 is now complete.

Claim 4. *In view of* ***A.3****, one actually has* $x^* \in \mathcal{C}_r^f(\mathcal{E})$.

The proof is very similar to the one used for proving that, under such a continuity assumption, an Edgeworth equilibrium of an economy is actually an element of the fuzzy core. By way of contradiction, assume that $\overline{x} \notin \mathcal{C}_r^f(\mathcal{E})$, thus that there exists $t = t^1 + t^2 \in [0,1]^I$, $t \neq 0$ and $x \in \prod_{i\in I} X_i$ such that $\sum_{i\in I} t_i x_i = \sum_{i\in I} t_i^1 e_i + \sum_{i\in I} t_i^2 \overline{x}_i$ and $u_i(x_i) > u_i(\overline{x}_i)$ $\forall i \in \text{supp } t$. In view of **A.3**, each $P_i(\overline{x}_i)$ is open in X_i. Thus, let $\varepsilon > 0$ be such that $1-\varepsilon < \lambda < 1 \implies \lambda x_i + (1-\lambda)e_i \in P_i(\overline{x}_i)$ and $\lambda x_i + (1-\lambda)\overline{x}_i \in P_i(\overline{x}_i)$ $\forall i \in \text{supp } t$. Let $s = s^1 + s^2 \in [0,1]^I$ be such that $s^1 \in Q^I$, $s^2 \in Q^I$, $t_i = 0 \implies s_i = 0$ and $t_i^1 > 0 \implies 1-\varepsilon < \frac{t_i^1}{s_i^1} < 1$, $t_i^2 > 0 \implies 1-\varepsilon < \frac{t_i^2}{s_i^2} < 1$. Set for each $i \in \text{supp } t$,

$$x_i^s = \frac{1}{s_i^1+s_i^2}\Big[s_i^1\big(\frac{t_i^1}{s_i^1}x_i + (1-\frac{t_i^1}{s_i^1})e_i\big) + s_i^2\big(\frac{t_i^2}{s_i^2}x_i + (1-\frac{t_i^2}{s_i^2})\overline{x}_i\big)\Big].$$

It is easily verified that $\sum_{i\in I}(s_i^1+s_i^2)x_i^s = \sum_{i\in I} s_i^1 e_i + \sum_{i\in I} s_i^2 \overline{x}_i$ and that $x_i^s \in P_i(\overline{x}_i)$ $\forall i \in \text{supp } s$, which contradicts $\overline{x} \in \mathcal{C}_r^e(\mathcal{E})$ and completes the proof of the proposition. ■

3. Decentralizing elements of the fuzzy rejective core

Proposition 3.1. *Under Assumptions* ***A.1****,* ***A.2****, let* $\overline{x} \in \mathcal{C}_r^f(\mathcal{E})$*. If* $N(\overline{x}) \neq \emptyset$*, then there exists a nonnull* $\overline{p} \in \mathbb{R}^\ell$ *such that* $(\overline{p}, \overline{x})$ *is a quasiequilibrium with dividends of the economy* $\mathcal{E}$*. If* $N(\overline{x}) = \emptyset$*,* $\overline{x}$ *is trivially the allocation of a quasiequilibrium with dividends.*

Proof. For sake of completeness of this paper, we repeat here the proof given by Konovalov (2005). Let us consider the convex sets $G_i^e = \{y_i - e_i : y_i \in P_i(\overline{x}_i)\}$, $G_i^{\overline{x}} = \{y_i - \overline{x}_i : y_i \in P_i(\overline{x}_i)\}$ and define

$$G = \text{co}\bigcup_{i\in I}(G_i^e \cup G_i^{\overline{x}}).$$

We first prove that $0 \notin G$. Indeed, if not, there exist for each $\in I$, $t_i^1 \geq 0$, $t_i^2 \geq 0$, and for each $i : t_i^1 > 0$, $y_i^1 \in P_i(\overline{x}_i)$, for each $i : t_i^2 > 0$, $y_i^2 \in P_i(\overline{x}_i)$, such that $\sum_{i\in I}(t_i^1 + t_i^2) = 1$ and

$$0 = \sum_{i\in I} t_i^1(y_i^1 - e_i) + \sum_{i\in I} t_i^2(y_i^2 - \overline{x}_i).$$

From this, one deduces:

$$\sum_{i\in I}(t_i^1+t_i^2)\Big[\frac{t_i^1}{t_i^1+t_i^2}y_i^1 + \frac{t_i^2}{t_i^1+t_i^2}y_i^2\Big] = \sum_{i\in I} t_i^1 e_i + \sum_{i\in I} t_i^2 \overline{x}_i$$

which, in view of Assumption **A.2**, contradicts $\overline{x} \in \mathcal{C}_r^f(\mathcal{E})$.

Let us now assume that $N(\overline{x}) \neq \emptyset$. Then, $G \neq \emptyset$ and there exist $\overline{p} \neq 0$ such that $\overline{p}\cdot g \geq 0$ $\forall g \in G$. From

$$y_i \in P_i(\overline{x}_i) \implies [\overline{p}\cdot y_i \geq \overline{p}\cdot e_i \text{ and } \overline{p}\cdot y_i \geq \overline{p}\cdot \overline{x}_i],$$

setting for each $i \in I$, $m_i = \max\{0, \overline{p} \cdot \overline{x}_i - \overline{p} \cdot e_i\}$, it is easily seen that $(\overline{p}, \overline{x})$ is a quasiequilibrium with dividends.

If $N(\overline{x}) = \emptyset$, setting for $m = (m_i)_{i \in I}$ any element of $\mathbb{R}^I_+$, $(0, \overline{x})$ is a quasiequilibrium with dividends. But for any $\overline{p}$, $(\overline{p}, \overline{x})$ is also a quasiequilibrium with dividends, letting for each $i \in I$, $m_i = \max\{0, \overline{p} \cdot \overline{x}_i - \overline{p} \cdot e_i\}$. ■

Propositions 2.2 and 3.1 have as common consequence the following result which extends for an exchange economy Theorem 3 in Le Van–Minh (2007).

Corollary 3.1. *Under Assumptions **A.1** – **A.5**, the economy $\mathcal{E}$ has a quasiequilibrium with dividends. In view of **A.4**, this quasiequilibrium is actually an equilibrium with dividends.*

Two last remarks are in order.

Remark 3.2. Given $x^* \in \mathcal{C}^f_r(\mathcal{E})$, it is possible to find $p^* \neq 0$ such that (x^*, p^*) is a quasiequilibrium with minimal dividends. Indeed, let $S = \{p \in \mathbb{R}^\ell : \|p\| = 1\}$. Recalling the proof of Proposition 3.1, if the set G is nonempty, the nonempty set $\{p \in S : \overline{p} \cdot g \geq 0 \ \forall g \in G\}$ is closed, thus compact, and there exists p^* which minimizes on this set the sum $\sum_{i \in I} \max\{0, p \cdot x_i^* - p \cdot e_i\}$. If $G = \emptyset$, there exists p^* which minimizes on S the sum $\sum_{i \in I} \max\{0, p \cdot x_i^* - p \cdot e_i\}$.

Remark 3.3. According to Definition 1.1, a (Walrasian) quasiequilibrium is a quasiequilibrium with dividends (x^*, p^*) such that $p^* \neq 0$ and all dividends m_i^* are equal to 0. If for each $i \in I$, $x_i^* \notin S_i$, in view of the strict quasi-concavity of utility functions, both conditions are obviously satisfied. Under stronger assumptions on the economy than the ones used in this paper, Allouch–Le Van (2009) have stated a weaker non-satiation condition for the existence of a (Walrasian) quasiequilibrium. In the next section, we formulate a weak non-satiation assumption, sufficient for quasiequilibrium existence, especially adapted to the exchange economies considered in this paper, that is to economies where consumers' utility functions are not assumed to be upper semicontinuous on their consumption set.

4. Existence of (Walrasian) quasiequilibrium under a weak non-satiation assumption

We consider on $\mathcal{E}$ the assumptions **A.1** – **A.3**, **A.5** and **A.6**, still assume without loss of generality that for each $i \in I$, $e_i \notin S_i$, and add the following weak non-satiation assumption:

A.6: For each consumer $i \in I$, if $S_i \cap \mathcal{A}_i \neq \emptyset$ then for all $x_i \in S_i$, $\lambda_i(x_i) := \inf\{\lambda > 0 : e_i + \lambda(x_i - e_i) \in S_i\} < 1$.

Note that for each i, the set of satiated points S_i is convex and would be closed if the utility function u_i was supposed to be upper semicontinuous. Assumption **A.6** is thus inconsistent with any assumption of upper semicontinuity of consumers' utility functions.

Let us define on X_i, the following utility functions:

$$\widehat{u}_i(x_i) = \begin{cases} u_i(x_i) & \text{if } x_i \notin S_i \\ u_i(x_i) + e^{-d(e_i, x_i)} & \text{if } x_i \in S_i \end{cases}$$

and consider the economy $\widehat{\mathcal{E}}$, obtained from $\mathcal{E}$ replacing by $\widehat{u}_i$ the consumers' utility functions u_i.

For proving equilibrium existence in $\mathcal{E}$, the strategy is different from the one used in Sections 2 and 3. We first establish the non-emptiness of the fuzzy core of $\widehat{\mathcal{E}}$ and thus the existence of a

(Walrasian) quasi-equilibrium for $\widehat{\mathcal{E}}$. The equilibrium existence for $\mathcal{E}$ will then follows from the equilibrium existence for $\widehat{\mathcal{E}}$.

Proposition 4.1. *Under the assumptions* ***A.1*** *–* ***A.3****,* ***A.5*** *and* ***A.6****,* $\mathcal{C}(\widehat{\mathcal{E}}) \neq \emptyset$.

Proof. It is easily verified that the functions $\widehat{u}_i \colon X_i \to \mathbb{R}$ are quasi-concave. We now prove that the set

$$\widehat{\mathcal{U}} = \{v = (v_i)_{i\in I} \colon \exists x = (x_i)_{i\in I} \in \mathcal{A}(\mathcal{E}), \text{ s.t. } 0 \le v_i \le \widehat{u}_i(x_i) \ \forall i \in I\}$$

is compact. Indeed let v^ν and $x^\nu \in \mathcal{A}(\mathcal{E})$ be such that for each $i \in I$, $\widehat{u}_i(e_i) = u_i(e_i) \le v_i \le \widehat{u}_i(x_i^\nu)$. Since $\mathcal{U}$ is strongly compact, there exist a subsequence x^{ν_k} of x^ν and an allocation $y \in \mathcal{A}(\mathcal{E})$ such that

$$\forall i \in I, \quad u_i(y_i) \ge \lim_{k\to\infty} u_i(x_i^{\nu_k}) \tag{4.1}$$

and one of the three cases stated in Definition 2.1 occur. From (4.1) and the definition of $\widehat{u}_i$, we first deduce that for each $i \in I$, the sequence $v_i^{\nu_k}$ converges to $v_i \ge 0$.

If $y_i \notin S_i$, then for k large enough, $x_i^{\nu_k} \notin S_i$. Then, from (4.1) and from $v_i^{\nu_k} \le \widehat{u}_i(x_i^{\nu_k}) = u_i(x_i^{\nu_k})$, we deduce: $v_i \le u_i(y_i) = \widehat{u}_i(y_i)$,

If $y_i \in S_i$, the third case may still occur. Then from (4.1), and from $v_i^{\nu_k} \le \widehat{u}_i(x_i^{\nu_k}) = u_i(x_i^{\nu_k})$, we deduce: $v_i \le u_i(y_i) \le \widehat{u}_i(y_i) =$, In the first case, we can assume without loss of generality $\lim_{k\to\infty} \|x_i^{\nu_k}\| = \infty$, so that from (4.1) and from $v_i^{\nu_k} \le \widehat{u}_i(x_i^{\nu_k}) \le u_i(x_i^{\nu_k}) + e^{-d(e_i, x_i^{\nu_k})}$, we deduce $v_i \le u_i(y_i) \le \widehat{u}_i(y_i)$. In the second case, the third one being excluded, we can assume without loss of generality $\widehat{u}_i(x_i^{\nu_k}) = u_i(x_i^{\nu_k}) + e^{-d(e_i, x_i^{\nu_k})}$, so that by using (4.1) and passing to limit in the relations $v_i^{\nu_k} \le \widehat{u}_i(x_i^{\nu_k})$, we get $v_i \le u_i(y_i) + d(e_i, y_i) = \widehat{u}_i(y_i)$. The proof of compactness of $\widehat{\mathcal{U}}$ is thus complete.

In view of Theorem 3.1 in Allouch–Florenzano (2004), the economy $\widehat{\mathcal{E}}$ has an Edgeworth equilibrium $\overline{x} = (\overline{x}_i)_{i\in I}$. We now prove that, under the assumption **A.6**, $\overline{x}$ is an element of the fuzzy core of $\widehat{\mathcal{E}}$. To see that, assume by contraposition that there exist $t = (t_I)_{I\in I} \in [0,1] \setminus \{0\}$ and $x \in \prod_{i\in I} X_i$ such that $\sum_{i\in I} t_i x_i = \sum_{i\in I} t_i e_i$ and $\widehat{u}_i(x_i) > \widehat{u}_i(\overline{x}_i) \ \ \forall i \in \text{supp } t$.

We first claim that there exists $\varepsilon > 0$ such that for each $i \in I$, $1-\varepsilon < \lambda < 1 \implies \widehat{u}_i\big(e_i + \lambda(x_i - e_i)\big) > \widehat{u}_i\big(\overline{x}_i\big)$.

For each $i \in I$, several cases must be considered:

- If $\overline{x}_i \in S_i$, from which it follows that $x_i \in S_i$ and $d(e_i, x_i) < d(e_i, \overline{x}_i)$, it suffices to take $1 - \varepsilon > \lambda_i(x_i)$.
- If $\overline{x}_i \notin S_i$ and $x_i \in S_i$, from which it follows that $u_i(x_i) + e^{-d(e_i,x_i)} > u_i(\overline{x}_i)$, it suffices to take $1 - \varepsilon > \lambda_i(x_i)$ to get for $\lambda > 1-\varepsilon$, $\widehat{u}_i\big(e_i + \lambda(x_i - e_i)\big) > u_i(x_i) + e^{-d(e_i,x_i)} = \widehat{u}_i(x_i)$.
- If $\overline{x}_i \notin S_i$ and $x_i \notin S_i$, from which it follows that $u_i(x_i) > u_i(\overline{x}_i)$, using **A.3**, it suffices to take $1-\varepsilon$ such that for $\lambda > 1-\varepsilon$, $\widehat{u}_i\big(e_i + \lambda(x_i - e_i)\big) \ge u_i\big(e_i + \lambda(x_i - e_i)\big) > u_i(\overline{x}_i) = \widehat{u}_i(\overline{x}_i)$.

We now classically define for each $i \in I$, $s_i \in Q$ such that $t_i = 0 \implies s_i = 0$ and $t_i > 0 \implies 1-\varepsilon < \frac{t_i}{s_i} < 1$ and $x_i^s = e_i + \frac{t_i}{s_i}(x_i - e_i)$. If $s = (s_i)_{i\in I}$, it is easily verified that $\sum_{i\in I} s_i x_i^s = \sum_{i\in I} s_i e_i$ and $\widehat{u}_i(x_i^s) > \widehat{u}_i(\overline{x}_i) \ \ \forall i \in \text{supp } s$, which contradicts the assumption that $\overline{x}$ is an Edgeworth equilibrium of $\widehat{\mathcal{E}}$. ■

Proposition 4.2. *Under the assumptions* ***A.1*** *–* ***A.3****,* ***A.5*** *and* ***A.6****, the economy* $\widehat{\mathcal{E}}$ *has a quasiequilibrium.*

Proof. Let $\overline{x}$ be the element of $\mathcal{C}(\widehat{\mathcal{E}})$ obtained in the previous proposition. As well known, if for all $i \in I$, $\overline{x}_i \in \mathrm{cl}\, \widehat{P}_i(\overline{x}_i)$, then there exists $\overline{p} \in \mathbb{R}^\ell$, $\overline{p} \neq 0$ such that $(\overline{x}, \overline{p})$ is a quasiequilibrium of $\widehat{\mathcal{E}}$. Recall that $\widehat{P}_i(\overline{x}_i) = \{x_i \in X_i \colon \widehat{u}_i(x_i) > \widehat{u}_i(\overline{x}_i)\}$. If $\overline{x}_i \in S_i$, the above condition follows from the definition of $\widehat{u}_i$. If $\overline{x}_i \notin S_i$, the above condition follows from Assumption **A.2**. ■

Corollary 4.1. *Under the assumptions* ***A.1*** *–* ***A.3***, ***A.5*** *and* ***A.6***, *the economy $\mathcal{E}$ has a quasiequilibrium. If we assume in addition* ***A.4***, *this quasiequilibrium is actually an equilibrium.*

Proof. A quasiequilibrium of $\widehat{\mathcal{E}}$ is obviously a quasiequilibrium of $\mathcal{E}$. Proving the second part of the statement of the corollary is standard. ■

Appendix

Proposition 1. *Assume Assumptions* ***A.1*** *–* ***A.4*** *on the economy $\mathcal{E}$. If $x^* \in \mathcal{A}_{IR}(\mathcal{E}) \setminus \mathcal{C}^e_r(\mathcal{E})$ then there exists τ^1, τ^2 in $([0,1] \cap \mathbb{Q})^I$ and $y \in \prod_{i \in I} X_i$ such that $\tau = \tau^1 + \tau^2 \neq 0$ and*

(a) $\sum_{i \in I} \tau_i y_i = \sum_{i \in I} \tau^1_i e_i + \sum_{i \in I} \tau^2_i x^*_i$,

(b) $u_i(y_i) > u_i(x^*_i)\ \forall i \colon \tau_i > 0$,

with $\tau_i > 0$ if and only if $i \notin S(x^*)$.

Proof. Indeed, let us assume that $x^* \in \mathcal{A}_{IR}(\mathcal{E}) \setminus \mathcal{C}^e_r(\mathcal{E})$ thus that

(a) $\sum_{i \in I} t_i z_i = \sum_{i \in I} t^1_i e_i + \sum_{i \in I} t^2_i x^*_i$,

(b) $u_i(z_i) > u_i(x^*_i)\ \forall i \colon t_i > 0$,

for some t^1, t^2 in $([0,1] \cap \mathbb{Q})^I$, $t = t^1 + t^2 \neq 0$ and $z \in \prod_{i \in I} X_i$.

We first claim that there exist $z' \in \prod_{i \in I} \mathrm{int}\, X_i$ and t'^1, t'^2 in $([0,1] \cap \mathbb{Q})^I$, $t' = t'^1 + t'^2 \neq 0$ such that

(a) $\sum_{i \in I} t'_i z'_i = \sum_{i \in I} t'^1_i e_i + \sum_{i \in I} t'^2_i x^*_i$,

(b) $u_i(z'_i) > u_i(x^*_i)\ \forall i \colon t'_i > 0$.

Indeed, it suffices to choose $\lambda \in Q \cap (0,1)$ such that if $z'_i = e_i + \lambda(z_i - e_i)$ then $u_i(z'_i) > u_i(x^*_i)\ \forall i \colon t_i > 0$. Then $\sum_{i \in I} \frac{t_i}{\lambda} z'_= \sum_{i \in I} \big(t^1_i + t_i(\frac{1}{\lambda})\big) e_i + \sum_{i \in I} t^2_i x^*_i$. As it follows from the convexity of X_i (**A.1**) and **A.4** that $z'_i \in \mathrm{int}\, X_i$, then $z' = (z'_i)_{i \in I}$ and, up to an obvious normalization, $t' = (\frac{t_i}{\lambda})_{i \in I}$ prove our claim.

To prove Proposition 1, starting now from the relations obtained in our first claim, we assume that for some i^0, $t'_{i^0} = 0$ and $i^0 \in N(x^*)$. Let y_{i^0} be such that $u_{i^0}(y_{i^0}) > u_{i^0}(x^*_{i^0})$. It follows from **A.2** that $u_{i^0}\big(x^*_{i^0} + \mu(y_{i^0} - x^*_{i^0})\big) > u_{i^0}(x^*_{i^0})$ for every $\mu \in (0,1]$. Letting $z'_{i^0} = x^*_{i^0} + \mu(y_{i^0} - x^*_{i^0})$, we now have: $\sum_{i \in I} t'_i z'_i + z'_{i^0} - \mu(y_{i^0} - x^*_{i^0}) = \sum_{i \in I} t'^1_i e_i + \sum_{i \in I} t'^2_i x^*_i + x^*_{i^0}$. Since for $t'_i > 0$, $z'_i \in \mathrm{int}\, X_i$, in view of **A.3**, it is possible to choose i^1 such that $t'_{i^1} > 0$ and $\mu \in (0,1]$ such that, if $z''_{i^1} = z'_{i^1} - \frac{\mu}{t'_{i^1}}(y_{i^0} - x^*_{i^0})$ then $u_{i^1}(z''_{i^1} > u_{i^1}(x^*_{i^1})$ and $\sum_{i \neq i^1} t'_i z'_i + t'_{i^1} z''_{i^1} + z'_{i^0} = \sum_{i \in I} t'^1_i e_i + \sum_{i \in I} t'^2_i x^*_i + x^*_{i^0}$. The same procedure can be repeated until there is no i such that $t'_i = 0$ and $i \in N(x^*)$. ■

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